



A guide to participating in CRUCIAL prediction markets

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1. Introduction

CRUCIAL is an initiative to use prediction markets, with expert participants, to elicit and aggregate the diverse expertise and knowledge of the participants and produce unified probability forecasts of climate-related risks. These forecasts can evolve as new information becomes available to participants.

Prediction markets are similar to financial markets, or recreational gambling, but they are not intended to transfer the ownership of assets of risks, or to provide entertainment, but are specifically designed to synthesize and summarize information from many sources.

Participants in CRUCIAL markets do not have to pay to take part or stake money. Instead, they are provided with credits with which to trade contracts that pay out more credits if an outcome associated with the contract occurs. The credits that participants accumulate can, however, be converted into cash rewards using funds provided by market sponsors. Through this mechanism it is hoped that prediction markets can become an alternative way of funding climate forecasting research in a performance-driven way.

This document is intended to provide guidance on how to trade in CRUCIAL's markets. If you are participating, then it is assumed you have relevant domain expertise in forecasting climate-related risks. The aim of this guide is to help you translate your own knowledge and expertise into effective trading strategies. By doing this you will also contribute to improving the collective predictions generated by the markets.

CRUCIAL uses a prediction market platform called AGORA that was specifically designed for climate-related applications.

2. How CRUCIAL markets are structured

2.1 Outcomes

Each CRUCIAL prediction market on the AGORA platform consists of a set of possible outcomes that are *mutually exclusive and comprehensively exhaustive* — that just means the outcomes are defined so that only one of them can occur but one of them *must* occur. For example, in a market to predict how many Atlantic hurricanes will occur in a particular year there might be 21 outcomes: no hurricanes, one hurricane, and so on up to “20 or more hurricanes”. Only one of these outcomes can occur and the inclusion of the open-ended final interval means that all eventualities are included.

Each outcome is assigned a price by the platform's “market maker” based on how many of that outcome the market maker has already sold (more about this later). The prices across all the outcomes in a market will always add up to one. Participants can buy one or more units of an outcome. When the actual outcome becomes known the market is



settled and all units of the correct outcome become worth one credit while all the other outcomes become worthless.

If the prevailing price of an outcome matches its expected value, then this price can be interpreted as the probability that this outcome will occur — according to the “market consensus”.

2.2 Contracts

On the AGORA platform you don’t trade individual outcomes directly. Instead, you can create *contracts* that consist of one or more outcomes. This feature is because some markets can have large numbers of outcomes (thousands in some cases) and having to trade each one individually would be inconvenient. For example, the tropical Pacific sea surface temperature anomaly might be partitioned into outcomes with each covering 0.1°C (with open-ended intervals to include any extremes). The contract feature allows you to create a single contract containing all outcomes in a range of your choosing: you could create a contract covering all outcomes larger than +2°C which you can trade as a single unit. The contracts you create are for your convenience, other participants cannot see them or trade them; they are just a handy way of trading multiple outcomes with the market maker.

Once you have created a contract the market maker will give it a price which is simply the sum of the prices of the outcomes it covers. This price is the “instantaneous” or “marginal” price, essentially the per unit price for buying an infinitesimally small number of the contract. If you place an order for a finite number of the contract the price per contract will increase, possibly quite a lot if it’s a large order. This will be explained further when we discuss how the market maker works.

2.3 Buying and selling contracts

Once you have bought a contract you can hold it until expiration — when the actual outcome is known, and the market is settled — or you can sell it back to the market maker at any time. To sell a contract you must have previously bought it. AGORA does not allow “shorting” (the selling of contracts that you don’t own). In financial markets shorting is a way to take advantage of overpricing (you borrow the asset and sell it in the hope you can buy it back at a lower price before you must return it to whoever loaned it to you). Because AGORA markets have all possible outcomes, and prices across these outcomes always sum to one, shorting is unnecessary: If you believe any outcomes are overpriced there must be other outcomes that are underpriced, so you can take advantage of the overpricing by buying the underpriced outcomes.

2.4 Joint-outcome markets

AGORA can support joint-outcome markets in which each outcome corresponds to values for two separate variables: e.g., temperature and rainfall, or carbon dioxide concentration and global temperature anomaly. The outcome space for these kinds of markets is a two-dimensional grid. The prices in a joint-outcome market provide an



implied joint-probability distribution. Although they will typically have many more outcomes than one-dimensional markets there is nothing fundamentally different about joint-outcome markets. The outcomes are still defined to be mutually exclusive and comprehensively exhaustive and have prices that sum to one.

3. The automated market maker

3.1 Why use an automated market maker?

CRUCIAL markets use an automated market maker (AMM) that is always willing to quote prices at which it will sell or buy back contracts. This contrasts with other prediction markets (and financial markets) that use a *continuous double auction* (CDA) that matches buyers and sellers directly. In a market with thousands of possible outcomes it might be very unlikely that two participants will want to trade the same collection of outcomes, making direct matching impossible. The AMM overcomes this and allows markets with large numbers of outcomes to function properly. Another feature of the AGORA AMM is that it is *subsidized*, it is willing to lose money in return for good information. This feature allows markets with small numbers of highly informed participants (and no “dumb money”) to work. The AMM essentially takes the role that uninformed participants play in other prediction markets.

3.2 How the market maker works

To trade you don't need to know the details of how the AMM is setting its prices, you just need to decide whether the price it is offering is above or below what you consider the “fair value” of the contract, which is the probability that one of the outcomes it includes will occur. However, the kind of people who participate in CRUCIAL's markets tend to be curious and knowledge of how the AMM works can help when it comes to constructing more sophisticated trading strategies, so this section describes the AMM that CRUCIAL's AGORA platform uses.

AGORA uses an AMM based on the logarithmic market scoring rule (LMSR). Let there be m outcomes (mutually exclusive and comprehensively exhaustive). Let q_i be the exposure of the AMM to the i th outcome ($i = 1, 2, \dots, m$), i.e., the number of that outcome it has sold to participants minus any that it has bought back. The AMM uses a cost function given by

$$C(\mathbf{q}) = b \log \left(\sum_{i=1}^m e^{q_i/b} \right)$$

where $\mathbf{q} = (q_1, q_2, \dots, q_m)$ and b is the *liquidity parameter*. The instantaneous, or marginal, price of outcome j is given by the derivative of this cost function w.r.t. to q_j .

$$p_j = \frac{\partial C(\mathbf{q})}{\partial q_j} = \frac{e^{q_j/b}}{\sum_{i=1}^m e^{q_i/b}}$$



from which we see that prices of all outcomes sum to one, $\sum_{i=1}^m p_i = 1$. We can also see that if one outcome becomes very popular relative to the others then its price will approach 1.0.

If we specify a contract as $\mathbf{w} = (w_1, w_2, \dots, w_m)$ the price that the AMM will ask for this contract is given by

$$C(\mathbf{q} + \mathbf{w}) - C(\mathbf{q}) = b \log \left(\sum_{i=1}^m e^{(q_i + w_i)/b} \right) - b \log \left(\sum_{i=1}^m e^{q_i/b} \right)$$

The parameter b controls how much the prices quoted by the AMM move for a given trade size. The larger the value of this parameter the less impact on prices a given trade will have.

3.3 Relationship with the logarithmic scoring rule

An elegant feature of the LMSR AMM (and where it gets its name) is that it rewards participants according to the logarithmic scoring rule for scoring probability forecasts. To see this consider the situation where there is a single participant who believes the true probabilities for each outcome are f_i ($i = 1, 2, \dots, m$) and they buy outcomes until the prices quoted by the AMM match these probabilities. The exposure of the AMM that corresponds to these prices is given by

$$q_j = b \log f_j + b \log \left(\sum_{i=1}^m e^{q_i/b} \right) = b \log f_j + bA$$

where $A = \log \left(\sum_{i=1}^m e^{q_i/b} \right)$.

Buying this exposure will cost the participant

$$C_f = b \log \left(e^A \sum_{i=1}^m f_i \right) - b \log m = bA - b \log m$$

If outcome k is the one that ultimately occurs, then the participant will receive q_k , so their net reward will be

$$q_k - C_f = b \log f_k + b \log m$$

which is linear in $\log f_k$, which is the logarithmic scoring rule.

The largest net reward the participant can receive is bounded by the case $f_k = 1$, and is given by $b \log m$. As mentioned, the AMM is subsidized and prepared to lose money, but its potential loss is controlled by the parameter b .



4. Trading strategies

4.1 Trading is driven by differences in opinion

The prices, p_i , for outcomes can be interpreted as the probabilities that they will occur according to the market consensus. If your own estimate of these probabilities, f_i , differs from the market consensus then there is an opportunity for you to trade with the expectation of making a profit.

Given the marginal prices, p_i , the AMM's exposure can be inferred using

$$q_i = b \log p_i + K$$

where K is some constant. The exposures can only be inferred up to some constant because uniform changes in exposure across all outcomes do not affect prices. If you buy one unit of every outcome it will always cost you one credit, and you are guaranteed to get that credit back at settlement.

Prices will move to match your own view of the probabilities if you make the following trade

$$w_i = b \log f_i - b \log p_i + K_w$$

Assuming you have no outcomes to sell you can make $w_i \geq 0$ by setting

$$K_w = - \min_i (b \log f_i - b \log p_i)$$

The cost of buying this position is given by

$$C_{p \rightarrow f} = b \log \left(\sum_{i=1}^m e^{(b \log p_i + K + b \log f_i - b \log p_i + K_w)/b} \right) - b \log \left(\sum_{i=1}^m e^{(b \log p_i + K)/b} \right) = K_w$$

So, your expected profit will be

$$\text{Expected profit} = \sum_{i=1}^m f_i w_i - K_w = b \left(\sum_{i=1}^m f_i \log f_i - \sum_{i=1}^m f_i \log p_i \right)$$

which is the liquidity parameter multiplied by the relative entropy between your probability distribution and the market distribution, which is kind of cool.

WHAT IF YOU ARE CASH CONSTRAINED?

i.e. MAXIMIZE EXPECTED PROFIT FOR A GIVEN OUTLAY

4.2 Arbitrage

Arbitrage is a term used in trading that refers to taking advantage of price discrepancies. The word is sometimes reserved for risk-free trades but is also frequently used for trades that aren't completely without risk: e.g., buying a commodity in one place more cheaply



than you can sell it in a different place — there is a risk that the logistics of transporting the commodity between these places might scupper the profitability of this trade. We will use the term arbitrage in this more general way to mean trades that exploit apparent inconsistencies in prices. For example, suppose in a market for the number of hurricanes an outcome is priced lower than the adjacent outcomes on either side: e.g., the price of 7 hurricanes is materially lower than both 6 or 8 hurricanes. Whatever your view on the likely number of hurricanes this bimodality seems unphysical and implies 7 is underpriced relative to its neighbours.

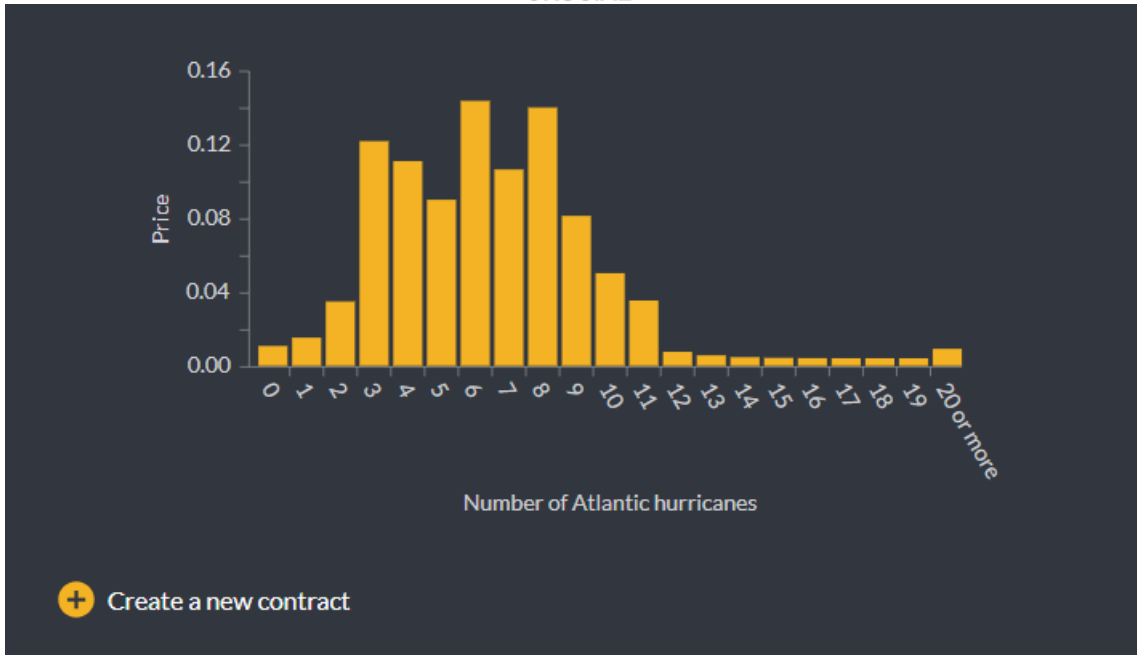
Another potential opportunity for arbitrage is when a strip of markets is predicting a quantity at consecutive horizons — e.g. the monthly tropical Pacific sea surface temperature anomaly (SSTA). While tropical Pacific SSTA does change from month to month its probability distribution at longer teams, at the limits of predictability, shouldn't change too much from one month to the next, and large differences between the price distributions for consecutive months.

It is possible that two CRUCIAL markets might predict *exactly* the same thing. This might seem odd but with joint-outcome markets it might be that two markets share one axis: e.g. a market to predict CO₂ concentration and global temperature anomaly and a market to predict CO₂ concentration and global sea-level rise. In this case it may be possible to engage in genuine risk-free arbitrage. In a joint-outcome market you can bet purely on one of the variables by creating a contract that covers a limited range of outcomes in that variable but all possible outcomes in the other variable. If the marginal distribution of prices for the same variable differs between markets, then there is an arbitrage opportunity that can be exploited by buying every outcome for that variable but buying each outcome in the market where it is cheapest. This will allow you to buy a complete set of outcomes — which is guaranteed to pay out 1.00 credit — for less than 1.00 credit.

5. Trading using the AGORA user interface

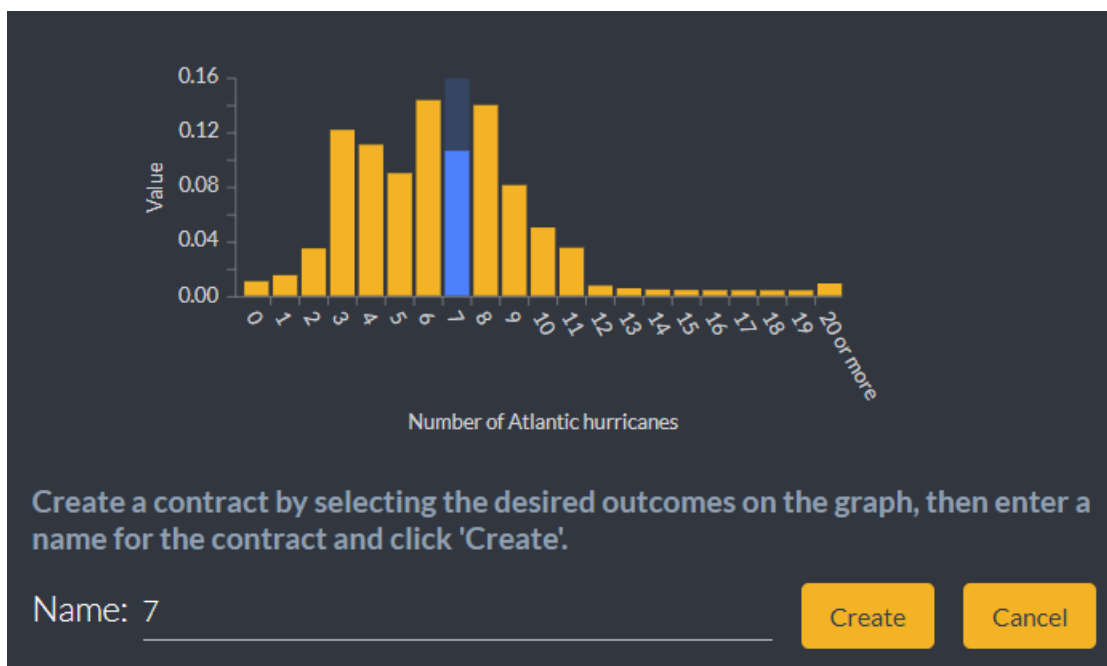
5.1 Outcomes and prices

When a market is selected in the AGORA user interface (UI) the current prices of each outcome are displayed. These are the instantaneous, or marginal prices, the prices for an infinitesimally small amount.



5.2 Creating and trading contracts

Outcomes cannot be traded directly, instead contracts are traded which can cover one or more outcomes. To create a contract select “create a new contract” and then select the outcome(s) you want to be included in this contract. AGORA will give the contract a default name, which you can change. For example, suppose we want to create a contract for 7 hurricanes.





Once the contract has been created it appears in our list of contracts and we can now trade it by entering an order. Suppose we want to buy 100 units of the contract, we enter +100 in the order column (or alternatively the quantity column can be modified).



The AMM quotes the price for these 100 contracts. Notice that, while the instantaneous price for this contract is 0.107, the AMM quotes a price of 0.109 per contract for 100 of them. The larger the order the greater the difference will be between the instantaneous price and the quoted price per contract. If the quoted price is acceptable, we can click buy and the contracts will be added to our inventory.

5.3 Multiple outcome contracts and baskets

Contracts can contain multiple outcomes. For example, we can define a contract covering a range of outcomes. Notice that we have changed the name of this contract to “between 4 and 10 hurricanes” rather than use the default name provided by AGORA. The outcomes included in a contract don’t have to be contiguous.



Value

Number of Atlantic hurricanes

Create a contract by selecting the desired outcomes on the graph, then enter a name for the contract and click 'Create'.

Name: Create Cancel

To sell contracts that we have already bought we just need to enter a negative order in the order column. The AMM will quote us the price at which it is willing to buy them back. We can also enter orders for different contracts, including a mix of buy and sells as below, which the AMM will price together as a “basket”. This allows us to execute all the trades together.

Price

Number of Atlantic hurricanes

Contract	Price	Quantity	Order	Action
7	0.112	50	-50	SELL 50 @ 0.110
between 4 and 10 hur...	0.727	50	+50	BUY 50 @ 0.730
Total:				BASKET: 30.928

+ Create a new contract